

AP Calculus AB

Implicit Differentiation

1) $x = \csc y$

$$1 = -\csc y \cot y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{-\csc y \cot y}$$

$$\frac{dy}{dx} = -\sin y \tan y$$

2) $x^5 + y^5 = 0$

a) $5x^4 + 5y^4 \cdot \frac{dy}{dx} = 0$

$$5y^4 \frac{dy}{dx} = -5x^4$$

$$\frac{dy}{dx} = -\frac{x^4}{y^4}$$

b) $\frac{dy}{dx} \Big|_{(3,-3)} = -\frac{(3)^4}{(-3)^4} = -1$

3) $y^3 = 27x$

a) $3y^2 \frac{dy}{dx} = 27$

$$\frac{dy}{dx} = \frac{27}{3y^2}$$

b) $\frac{dy}{dx} \Big|_{(1,3)} = 1$

4) $\cos(2y) = x$

a) $-2\sin(2y) \cdot \frac{dy}{dx} = 1$

$$\frac{dy}{dx} = -\frac{1}{2\sin(2y)}$$

b) $\frac{dy}{dx} \Big|_{(0, \frac{\pi}{2})} = -\frac{1}{2\sin(\frac{\pi}{2})} = -\frac{1}{2}$

5) $12x + 2y^2 = 8$

$$12 + 4y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{3}{y}$$

$$\frac{d^2y}{dx^2} = -\frac{y(0) - 3\frac{dy}{dx}}{y^2}$$

$$= \frac{3\frac{dy}{dx}}{y^2}$$

$$= \frac{3(-\frac{3}{y})}{y^2}$$

$$= -\frac{9}{y^3} = \boxed{-\frac{9}{y^3}}$$

6) $5y^8 + 7x^7 = 9y + 3x$

$$40y^7 \frac{dy}{dx} + 49x^6 = 9\frac{dy}{dx} + 3$$

$$40y^7 \frac{dy}{dx} - 9\frac{dy}{dx} = 3 - 49x^6$$

$$\frac{dy}{dx}(40y^7 - 9) = 3 - 49x^6$$

$$\frac{dy}{dx} = \frac{3 - 49x^6}{40y^7 - 9}$$

$$\boxed{\frac{dy}{dx} \Big|_{(1,1)} = -\frac{46}{31}}$$

$$7) x^2 + xy - y^2 = -1$$

$$2x + x \frac{dy}{dx} + y - 2y \frac{dy}{dx} = 0$$

$$\left. \frac{dy}{dx} \right|_{(3,5)} = \frac{-2(3)-5}{3-2(5)}$$

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = -2x - y$$

$$= \frac{-11}{-7} = \frac{11}{7}$$

$$\frac{dy}{dx}(x-2y) = -2x-y$$

$$\text{Tangent: } y-5 = \frac{11}{7}(x-3)$$

$$\frac{dy}{dx} = \frac{-2x-y}{x-2y}$$

$$\text{Normal: } y-5 = -\frac{7}{11}(x-3)$$

$$8) x^2 + y^2 = 5$$

$$x = -2$$

$$\text{point} \\ (-2)^2 + y^2 = 5$$

$$\text{slope}$$

$$2x + 2y \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

$$4 + y^2 = 5$$

$$y^2 = 1$$

$$y = \pm 1$$

$$(-2, 1)$$

$$(-2, -1)$$

$$\left. \frac{dy}{dx} \right|_{(-2,1)} = 2$$

$$\left. \frac{dy}{dx} \right|_{(-2,-1)} = -2$$

$$(-2, 1) \quad (-2, -1)$$

$$\text{Tangents: } y-1 = 2(x+2)$$

$$y+1 = -2(x+2)$$